

GCE AS/A level

0978/01

MATHEMATICS – FP2 Further Pure Mathematics

S15-0978-01

P.M. TUESDAY, 16 June 2015

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen. Answer **all** questions. Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question. You are reminded of the necessity for good English and orderly presentation in your answers. 1. (a) Express

 $\frac{5}{\left(x^2+1\right)\left(2-x\right)}$

in partial fractions.

(b) Using the substitution $u = \tan x$ and the result in (a), evaluate the integral

$$\int_0^{\frac{\pi}{4}} \frac{5}{2 - \tan x} \, \mathrm{d}x.$$

Give your answer correct to three significant figures.

2. The function *f* is defined by

$$f(x) = ax^3 + bx \qquad \text{for } x \le -1,$$

$$f(x) = x^2 - x + 2 \qquad \text{for } x > -1.$$

- (a) Given that *f* and its derivative are both continuous at x = -1, determine the values of the constants *a* and *b*. [6]
- (b) The equation f(x) = 0 has exactly one root. Determine its value. [2]
- **3.** The complex number $z = 2\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)$.
 - (a) Find the three cube roots of z, giving your answers in the form x + iy, with x and y correct to three decimal places. [6]
 - (b) Find the smallest positive integer n for which z^n is
 - (i) real,
 - (ii) imaginary. [3]
- 4. Find the general solution to the equation

$$\cos\left(\theta + \frac{\pi}{6}\right) + \cos\left(2\theta + \frac{\pi}{6}\right) + \cos\left(3\theta + \frac{\pi}{6}\right) = 0.$$
 [8]

5. Differentiate the following integrals with respect to *x*.

(a)
$$\int_0^x e^{\sqrt{u}} du$$
 [1]

(b)
$$\int_{0}^{x^{2}} e^{\sqrt{u}} du$$
 [3]
(c) $\int_{x}^{x^{2}} e^{\sqrt{u}} du$ [2]

[9]

[4]

- 6. The point P(x, y) moves in such a way that its distance from the point (0, 3) is equal to its distance from the line y + 3 = 0.
 - (a) Show that the locus of *P* is the curve *C* with equation $x^2 = 12y$. [2]
 - (b) (i) Show that the point $(6t, 3t^2)$ lies on C for all values of t.
 - (ii) Show that the equation of the tangent to C at the point $(6t, 3t^2)$ is

$$y = tx - 3t^2.$$

- (iii) Find the values of t for which the tangent passes through the point (0, -12).
- (iv) Hence find the angle between the two tangents to C from the point (0, -12). [9]
- 7. The function *f* is defined by

$$f(x) = \frac{1}{x-1} - \frac{4}{x-2}.$$

- (a) Write down the equations of the vertical asymptotes on the graph of *f*. [1]
- (b) Find the points of intersection of the graph of f with the coordinate axes. [3]
- (c) Find the coordinates of the stationary points on the graph of *f* and classify each point as a maximum or a minimum. [8]
- (d) Sketch the graph of f. [2]
- (e) The set S = [-1, 0]. Determine
 - (i) f(S),
 - (ii) $f^{-1}(S)$. [6]

END OF PAPER