## GCE AS/A level

0978/01


S15-0978-01

# MATHEMATICS - FP2 <br> Further Pure Mathematics 

P.M. TUESDAY, 16 June 2015

1 hour 30 minutes

## ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.


## INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.
Answer all questions.
Sufficient working must be shown to demonstrate the mathematical method employed.

## INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.
You are reminded of the necessity for good English and orderly presentation in your answers.

1. (a) Express

$$
\frac{5}{\left(x^{2}+1\right)(2-x)}
$$

in partial fractions.
(b) Using the substitution $u=\tan x$ and the result in (a), evaluate the integral

$$
\int_{0}^{\frac{\pi}{4}} \frac{5}{2-\tan x} \mathrm{~d} x
$$

Give your answer correct to three significant figures.
2. The function $f$ is defined by

$$
\begin{array}{ll}
f(x)=a x^{3}+b x & \text { for } x \leqslant-1, \\
f(x)=x^{2}-x+2 & \text { for } x>-1 .
\end{array}
$$

(a) Given that $f$ and its derivative are both continuous at $x=-1$, determine the values of the constants $a$ and $b$.
(b) The equation $f(x)=0$ has exactly one root. Determine its value.
3. The complex number $z=2\left(\cos \left(\frac{3 \pi}{4}\right)+\mathrm{i} \sin \left(\frac{3 \pi}{4}\right)\right)$.
(a) Find the three cube roots of $z$, giving your answers in the form $x+\mathrm{i} y$, with $x$ and $y$ correct to three decimal places.
(b) Find the smallest positive integer $n$ for which $z^{n}$ is
(i) real,
(ii) imaginary.
4. Find the general solution to the equation

$$
\begin{equation*}
\cos \left(\theta+\frac{\pi}{6}\right)+\cos \left(2 \theta+\frac{\pi}{6}\right)+\cos \left(3 \theta+\frac{\pi}{6}\right)=0 . \tag{8}
\end{equation*}
$$

5. Differentiate the following integrals with respect to $x$.
(a) $\int_{0}^{x} \mathrm{e}^{\sqrt{u}} \mathrm{~d} u$
(b) $\int_{0}^{x^{2}} \mathrm{e}^{\sqrt{u}} \mathrm{~d} u$
(c) $\int_{x}^{x^{2}} \mathrm{e}^{\sqrt{u}} \mathrm{~d} u$
6. The point $P(x, y)$ moves in such a way that its distance from the point $(0,3)$ is equal to its distance from the line $y+3=0$.
(a) Show that the locus of $P$ is the curve $C$ with equation $x^{2}=12 y$.
(b) (i) Show that the point $\left(6 t, 3 t^{2}\right)$ lies on $C$ for all values of $t$.
(ii) Show that the equation of the tangent to $C$ at the point $\left(6 t, 3 t^{2}\right)$ is

$$
y=t x-3 t^{2}
$$

(iii) Find the values of $t$ for which the tangent passes through the point $(0,-12)$.
(iv) Hence find the angle between the two tangents to $C$ from the point $(0,-12)$.
7. The function $f$ is defined by

$$
f(x)=\frac{1}{x-1}-\frac{4}{x-2} .
$$

(a) Write down the equations of the vertical asymptotes on the graph of $f$.
(b) Find the points of intersection of the graph of $f$ with the coordinate axes.
(c) Find the coordinates of the stationary points on the graph of $f$ and classify each point as a maximum or a minimum.
(d) Sketch the graph of $f$.
(e) The set $S=[-1,0]$. Determine
(i) $f(S)$,
(ii) $f^{-1}(S)$.

